

# Erosion Modeling for the Clive PA

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## 1.0 Erosion Model Input Distribution Summary

A summary of parameter values and distributions employed in the erosion modeling component of the Clive Performance Assessment (PA) model is provided in Table 1. Additional information on the derivation and basis for these inputs is provided in subsequent sections of this report.

For distributions, the following notation is used:

- $N(\mu, \sigma, [min, max])$  represents a normal distribution with mean  $\mu$  and standard deviation  $\sigma$ , and optional truncation at the specified *minimum* and *maximum*,
- $LN(GM, GSD, [min, max])$  represents a log-normal distribution with geometric mean  $GM$  and geometric standard deviation  $GSD$ , and optional *min* and *max*,
- $U(min, max)$  represents a uniform distribution with lower bound *min* and upper bound *max*,
- $Beta(\mu, \sigma, min, max)$  represents a generalized beta distribution with mean  $\mu$ , standard deviation  $\sigma$ , minimum *min*, and maximum *max*,
- $Gamma(\mu, \sigma)$  represents a gamma distribution with mean  $\mu$  and standard deviation  $\sigma$ , and
- $TRI(min, m, max)$  represents a triangular distribution with lower bound *min*, mode *m*, and upper bound *max*.

**Table 1. Summary of distributions for gully modeling**

GoldSim Model Parameter	Symbol	Units	Distribution or Value	Notes
Gully_b_parameter	$b$	—	normal( $\mu = -0.4, \sigma = 0.15$ , min = -0.75, max = -0.05 )	See Section 5.1
L_init	$L_0$	m	uniform( Small <sup>1</sup> , 5 )	See Section 4.1
AngleOfRepose_Gully	$\alpha_{gully}$	deg	normal( $\mu = 38, \sigma = 5$ , min = Small, max = 90 – Small )	Clover, 1998 (for gravel); See Section 4.1
AngleOfRepose_Fan	$\alpha_{fan}$	deg	uniform( 5, 10 )	See Section 4.1
Number_of_Gullies	—	—	Discrete uniform( min=1, max=20 )	See Section 4.0 and Section 6.0
ConvergenceCriterion	—	m <sup>3</sup>	0.01	modeling construct

## 2.0 Introduction

The safe storage and disposal of depleted uranium (DU) waste is essential for mitigating releases of radioactive materials and reducing exposures to humans and the environment. Currently, a radioactive waste facility located in Clive, Utah (the “Clive facility”) operated by the company EnergySolutions Inc. is being considered to receive and store DU waste that has been declared surplus from radiological facilities across the nation. The Clive facility has been tasked with

disposing of the DU waste in a manner that protects humans and the environment from future radiological releases.

To assess whether the proposed Clive facility location and containment technologies are suitable for protection of human health, specific performance objectives for land disposal of radioactive waste set forth in Utah Administrative Code (UAC) Rule R313-25 *License Requirements for Land Disposal of Radioactive Waste - General Provisions* must be met—specifically R313-25-8 *Technical Analyses*. In order to support the required radiological performance assessment (PA), a probabilistic computer model has been developed to evaluate the doses to human receptors and concentrations in groundwater that would result from the disposal of radioactive waste, and conversely to determine how much waste can be safely disposed at the Clive facility. The GoldSim systems analysis software (GTG, 2010) was used to construct the probabilistic PA model.

The site conditions, chemical and radiological characteristics of the wastes, contaminant transport pathways, and potential human receptors and exposure routes at the Clive facility that are used to structure the quantitative PA model are described in the conceptual site model documented in the white paper entitled *Conceptual Site Model for Disposal of Depleted Uranium at the Clive Facility* (Clive DU PA CSM.pdf).

The purpose of this white paper is to address specific details of the erosional processes that may affect cap performance and thus potentially result in the exposure of waste. This paper is organized to give a brief overview of erosional processes, present the overall modeling approach and assumptions, followed by the presentation of the mathematical formulae that are used to represent these processes in the GoldSim PA model.

### 3.0 Erosion Overview

Above-ground caps of waste repositories are subject to erosion by the forces of wind and water. The proposed waste disposal cell for DU at the Clive facility, which has an engineered above-ground cap, is subject to these erosional processes. Both wind and water erosion are represented in the Clive PA model. Wind erosion is briefly discussed below but is addressed in detail in the white paper, *Atmospheric Transport Modeling for the Clive Performance Assessment* (Atmospheric Modeling.pdf). Water erosion via the return of Lake Bonneville or a small lake is not discussed in this document, but is addressed in the Neptune white paper, *Deep Time Assessment for the Clive PA* (Deep Time Assessment.pdf). Other water erosional processes are described below.

The composition of the above-ground cap is an important factor in determining its erodibility. At the Clive facility, the top slope of the cap is composed of 18 inches of armor material (rip rap) above a 6-inch gravel layer, a 12-inch sacrificial soil layer, and a 6-inch lower gravel layer (EnergySolutions, 2009). The side slope has the same composition as the top slope, except with an 18-inch lower gravel layer. The large particle-sized material of the rip rap is generally considered to be resistant to movement by erosion. However, if there is sufficient disturbance by animals or OHVs, this may trigger erosion. The conceptual model for the Clive PA model assumes that wind-blown material will infill the pore space between the larger materials of the

cap, including the rip rap. This wind-blown material has a finer particle size and moves more readily with wind or water forces acting on the cap compared to the rip rap or gravel, which has larger particle sizes.

The following sections give conceptual overviews that relate to the various erosional processes that are considered as part of the Clive PA conceptual model. These sections set the conceptual basis for the modeling assumptions described in Section 5.0.

### 3.1 Wind Erosion

At the Clive facility, wind is expected to cause infilling of the spaces in the rip rap and the gravel layers over a relatively short period of time. For modeling simplification and because the overall time period of the model is so long, it is assumed that infill happens immediately at the beginning of the modeling run. Wind will also be the primary mechanism for blowing materials off-site. Once the cap infills with wind-blown material, it is assumed that the amount of soil removed off-site by wind erosion is the same as what is transported on-site, which results in a net mass balance of zero change by wind erosion. Details of wind erosion modeling for the Clive PA and the effects on dose to potential receptors are presented in the white paper, *Atmospheric Transport Modeling for the Clive Performance Assessment*.

### 3.2 Water Erosion

There are two types of water erosion in the Clive PA conceptual model: 1) sheet erosion and 2) gully erosion. These erosional processes are discussed in the following sections.

#### 3.2.1 Sheet Erosion

Sheet erosion is erosion of soil particles by water flowing overland as a “sheet” in a downslope direction. During extremely high rainfall events when rain falls faster than water can infiltrate, runoff can occur, which acts as a mechanism for removing/eroding cap materials. Sheet erosion is a uniform process over the area of the cap and depends largely on its slope, as well as rainfall intensity. This is different than erosion that flows in defined channels (i.e., Gully Erosion), which is discussed in Section 3.2.2.

In the central area, or top slope, of the embankment, where slopes are gradual (about 2% slope), sheet erosion would be slower than on the steeper side slopes of the cell (about 20% slope) (*Embankment Modeling for the Clive PA Model* white paper). As soil and loess move down slope by sheet erosion, it is likely that their volumes would be replenished by deposition of clean loess from the surrounding environs (i.e., a net balance of zero change). In the end, the total soil volume on the embankment would not change, though there would be a slow movement of soils down slope, along with the contaminants they could potentially contain if the cap were breached. However, sheet erosion is not included in the Clive PA model given that the top slope of the cap is nearly horizontal and that the side slope will not have DU waste buried under it in the current engineering design, so that not much contamination would likely be moved off site. Sheet erosion likely would have little effect, except possibly to move a small amount of potentially contaminated soil down slope. The potential contribution from sheet erosion could be evaluated



further by examining surface concentrations in the cap. Sheet erosion is not included in the Clive PA model since gully erosion is included and has a much more significant impact.

### 3.2.2 Gully Erosion

Gully erosion is a process that occurs when water flows in narrow channels, particularly during heavy rainfall events. Gully erosion typically results in a gully that has an approximate “V” cross section which widens (lateral growth) and deepens (vertical growth) through time until the gully stabilizes. The formation of gullies is a concern on uranium mill tailings sites and other long-term above-ground radioactive waste sites (NRC 2010). Gully erosion has the potential to move substantial quantities of both cap materials and waste, should the waste material be buried close to the surface. It occurs when surface water runoff becomes channeled and repeatedly removes soil along drainage lines, creating a fan of the removed materials.

There are two important features of the gully that need to be considered when modeling gully erosion: the thalweg and the angle of repose. The thalweg is a line that joins the lowest points of the gully along the entire length of the gully in its downward slope defining the gully’s deepest channel. It can conceptually be thought of as the bottom of the gully that runs along a downward slope. The angle of repose is the angle the side of the gully makes with the horizontal; it is a property of the material that is eroding.

The engineered cap at the Clive facility may be subject to gully erosion via a disturbance attributed to either an animal burrow or off-highway vehicle (OHV) track. It is assumed that a notch or nick will be created from these activities at some location on the surface of the cap and the feedback processes inherent in gully formation will cause erosion downward to the surrounding grade and erosion upward toward the top slope of the embankment. As water flows across the inner walls of the notch, erodible solid materials will be transported with it, creating a larger notch (both vertically and laterally) and thus a greater capacity to remove solid material. As this process continues, more material will erode down-gradient from the notch, as well as up-gradient from the notch. Also, as water flows down the thalweg it can undercut the gully banks, causing materials to slump into the thalweg, where they get washed along the downward slope until the angle of repose is reached. A wedge-shaped volume of material is removed and deposited on the neighboring flat as a sort of small alluvial fan, forming its own angle of repose. This process continues until the mouth of the gully has met the top of the removed material (Figure 1). With a brief screening assessment, gully erosion was evaluated as having the potential to occur at the Clive facility and is included using a simplified approach for the Clive PA model.

## 4.0 Gully Model Assumptions

In the development of the erosion modeling approach, Dr. Garry Willgoose, a geomorphologist with expertise in gully formation at The University of Newcastle, Australia, was consulted for advice relating to the modeling of gully formation at the Clive Site. Dr Willgoose is author of the erosion model SIBERIA (Willgoose, 2005) and has experience with gully formation on uranium mill tailings (Willgoose, 2010; Willgoose and Shermeen, 2006). The purpose of the initial gully

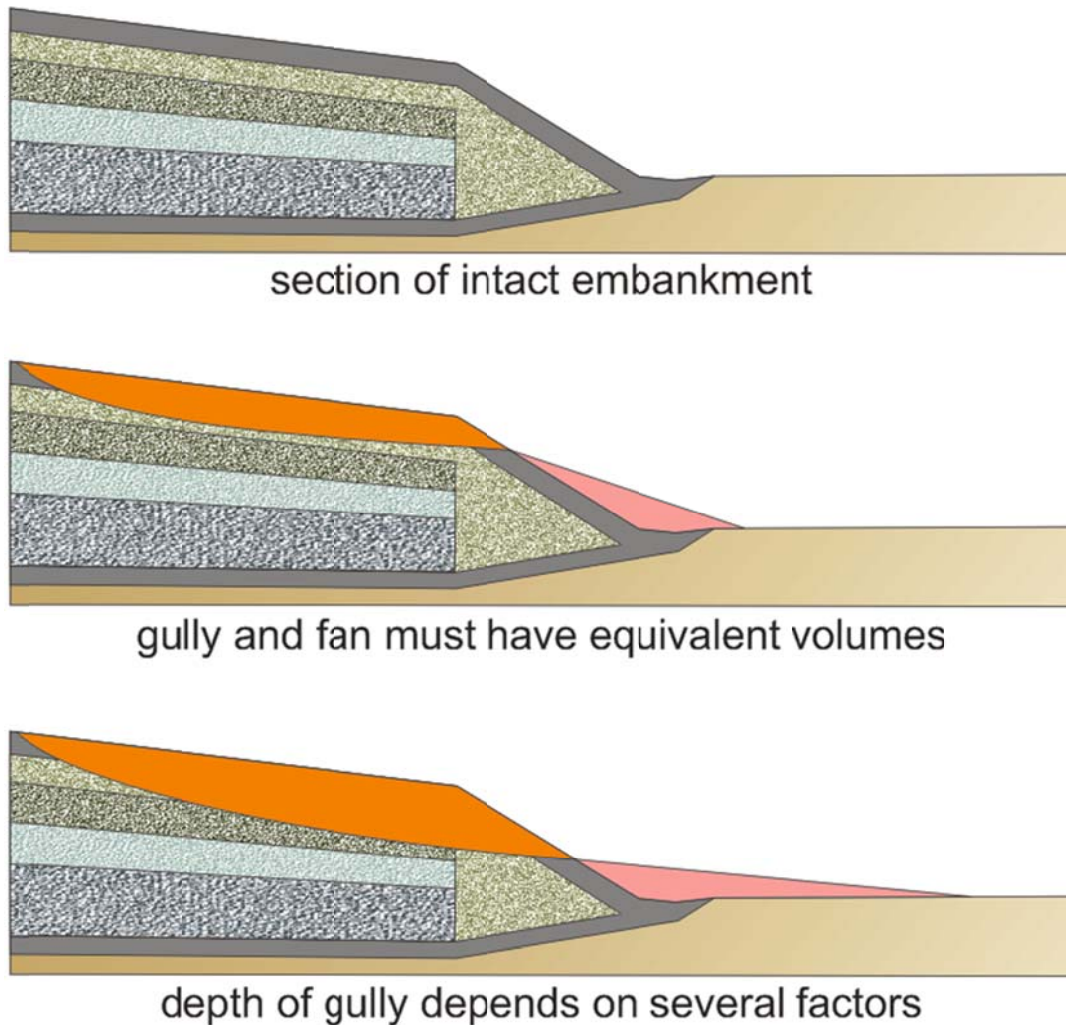
model in the Clive PA model is to determine whether gullies and fans are significant contributors to dose and whether a more sophisticated erosion model is needed. A simple screening-type gully model was developed with the advice of Dr. Willgoose. To that end, several simplifying assumptions are made:

- Gullies are assumed to form instantaneously, from the time of loss of institutional control. They do not evolve over time. To provide some understanding of what could happen if gullies were allowed to form at different times, concentrations in the gully material that is moved to the fan changes over time, as if the gully were formed instantaneously at any moment in time. These concentrations are used in the dose assessment. By this means, the effects of time are considered in the gully model.
- Gully formation occurs independently of main model processes. For example, processes such as biotic intrusion do not occur in gullies, nor does particle resuspension via wind erosion occur from the gully. In addition, the embankment remains intact – top and side slopes of the waste cell do not change in area or geometry with the formation of gullies.
- A small number of gullies are allowed to form, to evaluate the effects of more than one gully on dose and on model sensitivity. The distribution for the number of gullies allowed is a discrete uniform distribution from 1 to 20. Each gully has the same geometry for any given model realization.
- Types of gully-initiating events are not modeled. Conceptually, these could be either natural (e.g., animal burrows) or anthropogenic (e.g., OHV track). It is simply assumed that some triggering event occurs.
- The parameters for angles of repose, which are some of the parameters that dictate the geometry of the gullies, are based on the assumption of a homogenous cap material.
- The cross section of a gully is an inverted isosceles triangle, with the bottom vertex of the triangle following along a curved, downward sloping line that is the bottom of the gully (the thalweg).

As shown in Figure 1, gullies that form in the embankment may be of different depths or slightly different shapes. Thus, a different amount of material may be removed for different realizations, resulting in a different amount of potentially expose waste in different realizations. The first picture in Figure 1 shows the intact embankment, with different color shades demonstrating different layers of the cap and waste. The second picture in Figure 1 illustrates a shallow gully formed so that the gully and fan have equal volumes. It is clear that the height of the fan aligns with the mouth of the gully. The third picture in Figure 1 shows another, deeper gully formed. These gully depictions show the mouth of the gully and the height of the fan aligning, as well as equal volumes of fan and gully.

Gully geometry parameters are simulated probabilistically and are constant over a realization, assuming homogeneous materials. These parameters are then used to calculate the depth and volume of the formed gully. Based on this geometry, the amount of exposed waste is then calculated and included in the dose assessment as a soil concentration across the surface area of the fan and the gully.

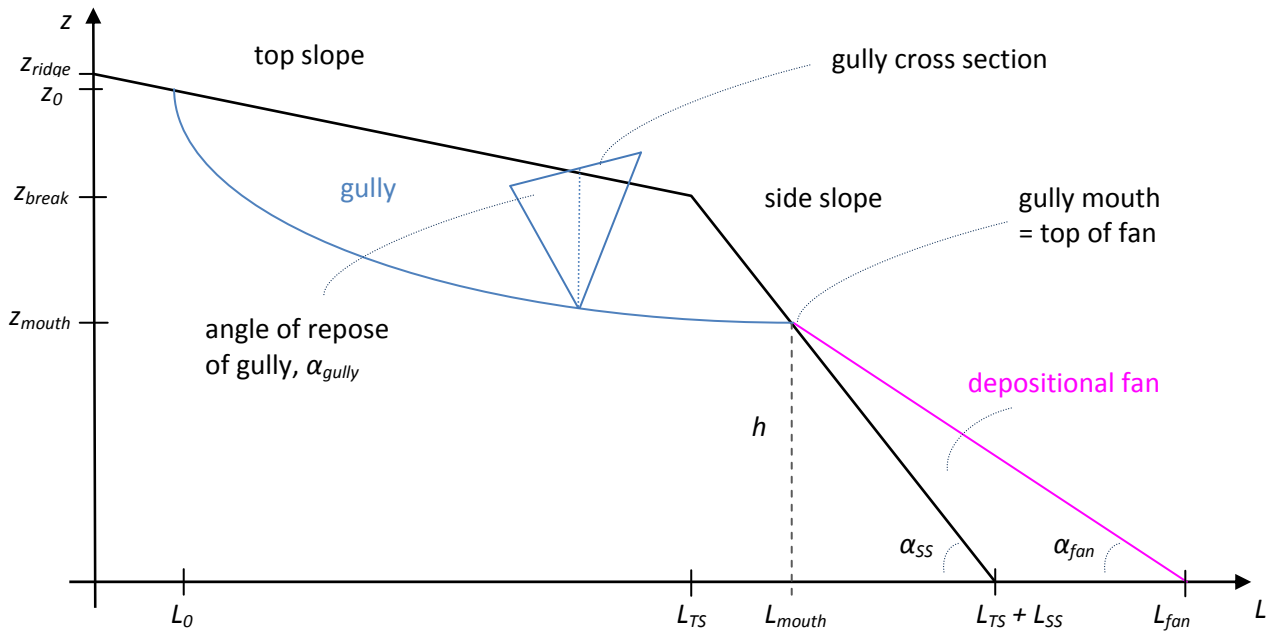
The remainder of this document describes gully geometry and the derivation of a simple model of a gully, based on recommendations from Dr. Willgoose, as well as the implementation of this model in the Clive PA GoldSim model.



**Figure 1. Illustration of the embankment with and without gullies**

#### **4.1 Gully Geometry Overview**

The overall geometry of the simplified gully used in the Clive PA GoldSim model is illustrated in Figure 2. Both Figure 1 and Figure 2 show a representation of half of the embankment, with the ridge of the top slope on the vertical axis. The vertical axis in Figure 2 is the elevation above ground surface, and the horizontal axis is the distance from the ridge of the embankment. The thalweg of the gully is the blue curved line in Figure 2; it forms the bottom of the gully, sloping downward toward the fan.



**Figure 2. Longitudinal cross-section of a gully in the embankment.**

Any cross section of the gully is assumed to be triangular, with the angle of repose of the gully being the angle that the gully makes with a horizontal line. The height of the thalweg when it comes out of the embankment through the side slope is also the height of the fan. This parameter,  $h$ , is also denoted as  $z_{mouth}$ , the elevation of the mouth of the gully. The break in slope is where the top slope and the side slope meet, denoted in Figure 2 by the point  $(L_{TS}, z_{break})$ .

The geometry of the gully is fully described by the engineering design of the cap, as described above, and by stochastic parameters for the angle of repose of the gully in the cap material ( $\alpha_{gully}$ ), the angle of repose of the eroded cap material ( $\alpha_{fan}$ ), the point of initiation of the gully on the cap ( $L_0$ ), and the shape parameter of the longitudinal cross section of the gully ( $b$ ). For these distributions, best professional judgment was used to create reasonably wide distributions that capture uncertainty in these parameters.

The angle of repose of the materials in the gully was represented by choosing values based on gravel, with a mean of 38 degrees, from an estimated range of 30 – 45 degrees (Clover, 1998). A standard deviation of 5 degrees was chosen to allow a slightly wider range of angles, since there is uncertainty in this parameter. Thus a normal distribution was assigned with a mean of 38 degrees, a standard deviation of 5 degrees, a minimum of near zero ( $1E-30$ ) and a maximum of near 90 degrees ( $90 - 1E-30$ ). The minimum and maximum were chosen by physical constraints.

The gully is assumed to begin less than 2 m from the ridge of the cap (Garry Willgoose, personal communication, 3 Jan 2011). As the point of initiation of the gully gets closer to the ridge, the slope of the gully approaches infinity, so the cap should not start at the ridge itself. A uniform distribution was assigned to  $L_0$ , ranging from near zero ( $1E-30$ ) to 5 m. A gully is not allowed to begin at  $L_0$  equal to zero, exactly at the ridge of the cap; rather, it is kept to one side of the ridge.

The angle of repose of the fan is limited on the high end by the angle created by the side slope and the ground surface, which is about 12 degrees. Since the fan partially lies on top of the side slope, the fan must form a smaller angle. As well, there is some limitation for the smallest angle this fan can form. Considering the large particle size of the gravel and rip rap, it is assumed that the minimum angle of the fan is 5 degrees. So the distribution for  $\alpha_{fan}$  is chosen as a uniform distribution from 5 degrees to 10 degrees.

The distribution for  $b$  is described in Section 5.2.

The notation for parameters in Figure 2 is used in the equations below. The following section describes geometry of the gullies as represented by the model and how the dimensions of the gullies are calculated.

## 5.0 Gully Calculations

The following subsections present the various mathematical formulae for calculating the components of the overall gully model.

### 5.1 Equation for thalweg elevation (gully bottom)

The following form for the slope of the thalweg of the gully as suggested by Dr. Garry Willgoose (personal communication, 3 Jan 2011) is:

$$Slope = \frac{dz_{gully}}{dL} = aL^b \quad (1)$$

where

- $z_{gully}$  is the height of the gully thalweg above natural ground surface (m)
- $L$  is the horizontal distance from the ridge of the cap downslope (m),
- $a$  is an amplitude parameter of the steepness of the thalweg slope (unitless), and
- $b$  is a power parameter (unitless), representing the curve of the thalweg.

Conditional on the value of  $b$ , the value of  $a$  can be calculated so that the elevation of the mouth of the gully matches the elevation of the fan of material that is washed out of the gully. In order to include the uncertainty in the model, a probability distribution was chosen to represent  $b$ . A mean value for  $b$  of -0.4 was estimated from the geomorphology of erosion profiles (Garry Willgoose, personal communication, 3 Jan 2011), and uncertainty about that value was implemented by representing  $b$  with a truncated normal distribution with a mean of -0.4 and a standard deviation of 0.15, truncated to be between -0.75 and -0.05.

Integrating each side of this equation results in an equation for  $z_{gully}$ , the height above ground surface of the thalweg along any point of the thalweg:

$$z_{gully} = \int aL^b dL \quad (2)$$

So,

$$z_{gully} = \frac{a}{b+1} L^{b+1} + C \quad (3)$$

where

$C$  is the constant of integration.

To find a value for  $C$ , the point of intersection of the gully and the top slope of the cap, i.e., where the gully begins, can be used. The top slope of the cap can be represented by the line

$$z_{TS} = \frac{(z_{break} - z_{ridge})}{L_{TS}} L + z_{ridge} \quad (4)$$

Setting  $z_{gully}$  equal to  $z_{TS}$  at the start of the gully, i.e., at  $L_0$ , yields

$$\frac{a}{b+1} L_0^{b+1} + C = \frac{(z_{break} - z_{ridge})}{L_{TS}} L_0 + z_{ridge} \quad (5)$$

Solving for  $C$ :

$$C = \frac{(z_{break} - z_{ridge})}{L_{TS}} L_0 + z_{ridge} - \frac{a}{b+1} L_0^{b+1} \quad (6)$$

Let

$$B_1 = -\frac{1}{b+1} L_0^{b+1} \quad (7)$$

and

$$B_0 = \frac{(z_{break} - z_{ridge})}{L_{TS}} L_0 + z_{ridge} = S_{TS} L_0 + z_{ridge} \quad (8)$$

where

$S_{TS}$  is the slope of the top slope of the cap.

Note that this expression,  $B_0$ , is the same as the height of the gully where the gully initiates.

Now there is an expression for the elevation of the bottom of the gully ( $z_{gully}$ ) in terms of the distance from the ridge of the cap:

$$z_{gully} = \frac{a}{b+1} L^{b+1} - B_1 a + B_0 \quad (9)$$

## 5.2 Solving for Gully Elevation

There are two sets of equations that are fundamental to solving this system. First, it is assumed that if a gully forms, it comes out of the side slope, so that the mouth of the bottom of the gully must intersect the line that forms the top of the side slope. In other words, the equation for the height of the bottom of the gully, evaluated where the gully emerges, must be equal to the elevation of the side slope, evaluated where the mouth of the gully emerges. Written mathematically, this becomes:

$$z_{gully} \Big|_{L_{mouth}} = z_{SS} \Big|_{L_{mouth}} \quad (10)$$

where

$z_{SS}$  is the elevation of the side slope at any distance  $L$  from the break to the ground surface (m).

The second key equation is that, the volume of cap materials removed by the gully must equal the volume of the material in the fan, following conservation of mass:

$$V_{gully} = V_{fan} \quad (11)$$

where

$V_{gully}$  is the volume of the gully in the cap ( $m^3$ ), and  
 $V_{fan}$  is the volume of the gully in the fan ( $m^3$ ).

In terms of the top slope and side slope, this equation can be written

$$V_{gully}^{TS} + V_{gully}^{SS} = V_{fan} \quad (12)$$

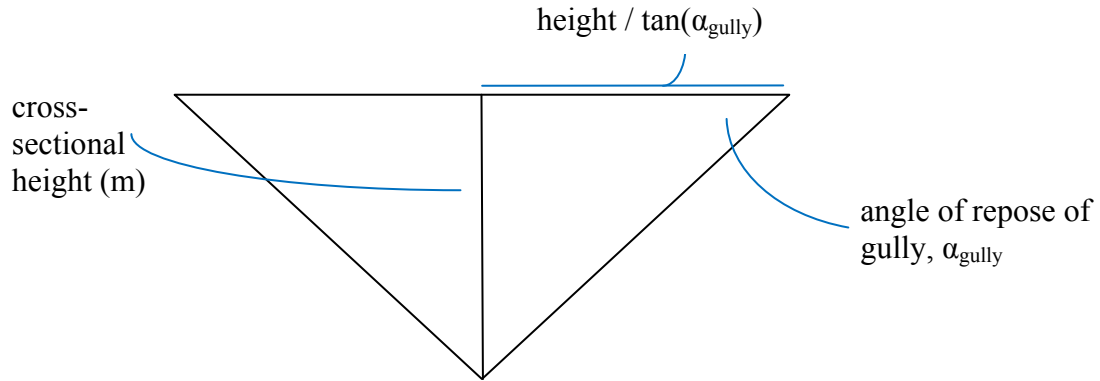
where

$V_{gully}^{TS}$  is the volume of the gully in the top slope of the cap ( $m^3$ ), and  
 $V_{gully}^{SS}$  is the volume of the gully in the side slope of the cap ( $m^3$ ).

These equations can be used to express all other unknown variables in terms of two variables:  $a$  and  $h$ , which is the elevation of the mouth of the gully. Using these key equations, the system of equations can be solved for  $a$  and  $h$ .

### 5.2.1 Volume of the Gully in the Top Slope of the Cap

As shown in Figure 2 and Figure 3, the cross-sectional area of the gully is assumed to be an isosceles triangle (Willgoose, personal communication, 3 Jan 2011). The height of the triangle is the difference between the height of the top slope and the height of the bottom of the gully. The angle of repose of the gully walls is the angle the gully makes with the horizontal. The base of the triangle is twice the height divided by the tangent of that angle.



**Figure 3. Cross-sectional view of gully**

The cross-sectional area of the gully ( $\text{m}^2$ ) can be represented by

$$A = \frac{1}{2} \text{base} \times \text{height} \quad (13)$$

$$A = \frac{\text{height} \times \text{height}}{\tan \alpha_{\text{gully}}} \quad (14)$$

and

$$A = \frac{(z_{TS} - z_{\text{gully}})^2}{\tan \alpha_{\text{gully}}} \quad (15)$$

$$A = \frac{(S_{TS}L + z_{\text{ridge}} - (\frac{a}{b+1}L^{b+1} - B_1a + S_{TS}L_0 + z_{\text{ridge}}))^2}{\tan \alpha_{\text{gully}}} \quad (16)$$

This equation simplifies to



$$A = \frac{(S_{TS}L - \frac{a}{b+1}L^{b+1} + B_1a - S_{TS}L_0)^2}{\tan \alpha_{gully}} \quad (17)$$

So, the volume of the gully in the top slope ( $m^3$ ) is the integral of the cross-sectional area from the initial point of the gully,  $L_0$  to the break between the top slope and the side slope, which corresponds to the length of the top slope,  $L_{TS}$ .

$$V_{gully}^{TS} = \int_{L_0}^{L_{TS}} \frac{(S_{TS}L - \frac{a}{b+1}L^{b+1} + (B_1a - S_{TS}L_0))^2}{\tan \alpha_{gully}} dL \quad (18)$$

Simplifying, the volume becomes

$$V_{gully}^{TS} = \frac{1}{\tan \alpha_{gully}} \int_{L_0}^{L_{TS}} (S_{TS}^2 L^2 - \frac{2aS_{TS}}{b+1} L^{b+2} + \frac{a^2}{(b+1)^2} L^{2b+2} + 2(B_1a - S_{TS}L_0)S_{TS}L - \frac{2a(B_1a - S_{TS}L_0)}{b+1} L^{b+1} + (B_1a - S_{TS}L_0)^2) dL \quad (19)$$

$$V_{gully}^{TS} = \frac{1}{\tan \alpha_{gully}} \left[ \frac{S_{TS}^2 L^3}{3} - \frac{2aS_{TS}L^{b+3}}{(b+1)(b+3)} + \frac{a^2 L^{2b+3}}{(b+1)^2(2b+3)} + (B_1a - S_{TS}L_0)S_{TS}L^2 - \frac{2a(B_1a - S_{TS}L_0)}{(b+1)(b+2)} L^{b+2} + (B_1a - S_{TS}L_0)^2 L \right]_{L_0}^{L_{TS}} \quad (20)$$

and finally,

$$V_{gully}^{TS} = \frac{1}{\tan \alpha_{gully}} \left[ \frac{S_{TS}^2 (L_{TS}^3 - L_0^3)}{3} - \frac{2aS_{TS}(L_{TS}^{b+3} - L_0^{b+3})}{(b+1)(b+3)} + \frac{a^2 (L_{TS}^{2b+3} - L_0^{2b+3})}{(b+1)^2(2b+3)} + (B_1a - S_{TS}L_0)S_{TS}(L_{TS}^2 - L_0^2) - \frac{2a(B_1a - S_{TS}L_0)}{(b+1)(b+2)} (L_{TS}^{b+2} - L_0^{b+2}) + (B_1a - S_{TS}L_0)^2 (L_{TS} - L_0) \right] \quad (21)$$

## 5.2.2 Volume of the Gully in the Side Slope of the Cap

The volume of the gully in the side slope is derived in a similar fashion to how the volume was derived for the gully in the top slope. The only differences are that the equation for the line made by the top of the side slope is used instead of the equation of the line made by top slope and that the limits of integration are from the edge of the top slope (the break) to the mouth of the gully, at an unknown value,  $L_{mouth}$ .

The side slope of the cap can be represented by the line

$$z_{SS} = -\frac{z_{break}}{L_{SS}} L + B_2 \quad (22)$$

where

$$B_2 = \frac{z_{break}}{L_{SS}} (L_{SS} + L_{TS}) \quad (23)$$

So, the volume of the gully in the side slope is the integral of the cross-sectional area of the gully in the side slope between the break ( $L_{TS}$ ) and the distance at which the gully mouth comes out the side slope ( $L_{mouth}$ ).

$$V_{gully}^{SS} = \int_{L_{TS}}^{L_{mouth}} \frac{\left(-\frac{z_{break}}{L_{SS}}L + B_2 - \left(\frac{a}{b+1}L^{b+1} + (B_0 - aB_1)\right)^2\right)}{\tan \alpha_{gully}} dL \quad (24)$$

$$V_{gully}^{SS} = \frac{1}{\tan \alpha_{gully}} \int_{L_{TS}}^{L_{mouth}} \left(-\frac{z_{break}}{L_{SS}}L - \frac{a}{b+1}L^{b+1} + (aB_1 - B_0 + B_2)\right)^2 dL \quad (25)$$

Simplifying, the volume becomes

$$V_{gully}^{SS} = \frac{1}{\tan \alpha_{gully}} \int_{L_{TS}}^{L_{gully}} \left(\frac{z_{break}^2}{L_{SS}^2}L^2 + \frac{2az_{break}}{L_{SS}(b+1)}L^{b+2} + \frac{a^2}{(b+1)^2}L^{2b+2} - 2\frac{z_{break}}{L_{SS}}(aB_1 - B_0 + B_2)L - \frac{2a(aB_1 - B_0 + B_2)}{b+1}L^{b+1} + (aB_1 - B_0 + B_2)^2\right) dL \quad (26)$$

$$V_{gully}^{SS} = \frac{1}{\tan \alpha_{gully}} \left[ \frac{z_{break}^2}{L_{SS}^2} \frac{L^3}{3} + \frac{2az_{break}}{L_{SS}(b+1)(b+3)}L^{b+3} + \frac{a^2L^{2b+3}}{(b+1)^2(2b+3)} - \frac{z_{break}}{L_{SS}}(aB_1 - B_0 + B_2)L^2 - \frac{2a(aB_1 - B_0 + B_2)}{(b+1)(b+2)}L^{b+2} + (aB_1 - B_0 + B_2)^2L \right] \Bigg|_{L_{TS}}^{L_{gully}} \quad (27)$$

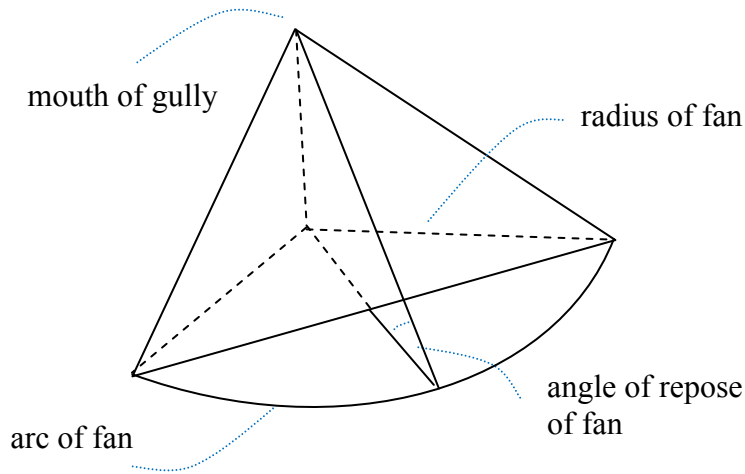
and finally,

$$V_{gully}^{SS} = \frac{1}{\tan \alpha_{gully}} \left[ \frac{z_{break}^2}{L_{SS}^2} \frac{(L_{gully}^3 - L_{TS}^3)}{3} + \frac{2az_{break}}{L_{SS}(b+1)(b+3)}(L_{gully}^{b+3} - L_{TS}^{b+3}) + \frac{a^2(L_{gully}^{2b+3} - L_{TS}^{2b+3})}{(b+1)^2(2b+3)} - \frac{z_{break}}{L_{SS}}(aB_1 - B_0 + B_2)(L_{gully}^2 - L_{TS}^2) - \frac{2a(aB_1 - B_0 + B_2)}{(b+1)(b+2)}(L_{gully}^{b+2} - L_{TS}^{b+2}) + (aB_1 - B_0 + B_2)^2(L_{gully} - L_{TS}) \right] \quad (28)$$

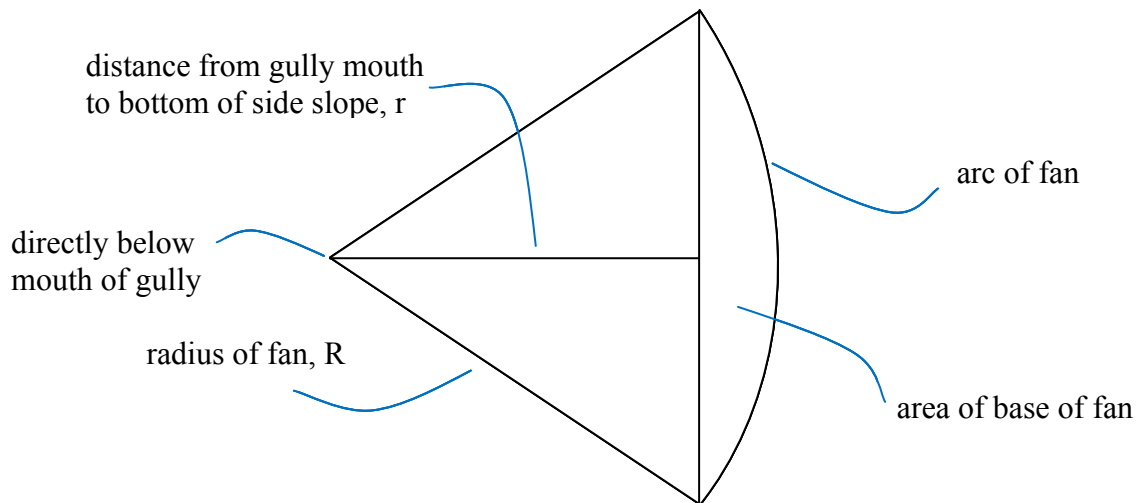
### 5.2.3 Volume of the Fan

The fan comes from the mouth of the gully, lies along the side slope, and continues to the ground surface (Figure 2). Figure 4 shows a 3-dimensional view of the fan. Figure 5 depicts a birds-eye view of the fan, looking through the fan to the bottom footprint of the fan. The base of the fan is the circular segment. The triangular area is the shadow of the part of the fan that lies on the

surface of the side slope. The apex of the fan represents the point of the bottom of the gully mouth.



**Figure 4. View of the fan in 3 dimensions.**



**Figure 5. Top view of the fan geometry, looking through the fan to the footprint.**

The fan is treated as a pyramidal structure. As such, the volume of the fan corresponds to  $1/3$  the area of the base multiplied by the height:

$$V_{fan} = \frac{1}{3} Area_{base} \times height \quad (29)$$

For this fan, the area of the base is the area of the circular segment (Figure 5):

$$Area_{base} = R^2 \cos^{-1}\left(\frac{r}{R}\right) - r\sqrt{R^2 - r^2} \quad (30)$$

where

- $R$  is the radius of the fan (m), and
- $r$  is the horizontal distance from the gully mouth to the bottom, or ground surface, of the side slope (m).

For more information on understanding this area calculation, see Weisstein, 2011a, for example.

The radius of the fan can be expressed in terms of the angle of repose of the fan, see Figure 2 and Figure 4.

$$R = \frac{h}{\tan \alpha_{fan}} \quad (31)$$

where

- $h$  is the height of the mouth of the gully (m), and
- $\alpha_{fan}$  is the angle of repose of the fan (deg).

Similarly, the distance,  $r$ , from the gully mouth to the outer edge of the side slope is

$$r = \frac{h}{\tan \alpha_{SS}} \quad (32)$$

where

- $\alpha_{SS}$  is the angle the side slope makes with the ground surface (deg).

Now the volume of fan can be expressed in terms of the area in Eq. (30), with new expressions for  $R$  and  $r$ , and the height to the mouth of the gully,  $h$ .

$$V_{fan} = \frac{h}{3} \left( \frac{h^2}{\tan^2 \alpha_{fan}} \cos^{-1}\left(\frac{\frac{h}{\tan \alpha_{SS}}}{\frac{h}{\tan \alpha_{fan}}}\right) - \frac{h}{\tan \alpha_{SS}} \sqrt{\frac{h^2}{\tan^2 \alpha_{fan}} - \frac{h^2}{\tan^2 \alpha_{SS}}} \right) \quad (33)$$

Simplifying yields

$$V_{fan} = \frac{h^3}{3} \left( \frac{1}{\tan^2 \alpha_{fan}} \cos^{-1} \left( \frac{\tan \alpha_{fan}}{\tan \alpha_{SS}} \right) - \frac{1}{\tan \alpha_{SS}} \sqrt{\frac{1}{\tan^2 \alpha_{fan}} - \frac{1}{\tan^2 \alpha_{SS}}} \right) \quad (34)$$

### 5.2.4 Expressing $a$ in terms of $h$

The components for Eq. (11) are now given by the volume of the gully in the top slope (Eq. (21)), the volume of the gully in the side slope (Eq. (28)), and the volume of the fan (Eq. (33)). Next, Eq. (10) can be expanded to express  $a$  in terms of  $h$ , the height of the gully.

$$\frac{a}{b+1} L_{mouth}^{b+1} - B_1 a + z_0 = -\frac{z_{break}}{L_{SS}} L_{mouth} + \frac{z_{break}}{L_{SS}} (L_{SS} + L_{TS}) \quad (35)$$

This equation can be solved for  $a$  so that

$$a = \frac{\frac{z_{break}}{L_{SS}} L_{mouth} - \frac{z_{break}}{L_{SS}} (L_{SS} + L_{TS}) + z_0}{B_1 - \frac{1}{b+1} L_{mouth}^{b+1}} \quad (36)$$

To express  $a$  in terms of  $h$ , an equation for  $L_{mouth}$  is used, based on Figure 2.

$$L_{mouth} = L_{SS} + L_{cap} - \frac{h}{\tan \alpha_{SS}} \quad (37)$$

Now there are sufficient equations to solve for  $h$ , and all other variables can be re-written as a function of  $h$ . The equation for the elevation of the gully bottom can then be computed at any point along the gully.

## 6.0 Implementation in GoldSim

The gully calculations presented above are used in the Clive PA GoldSim model to allow the formation of a gully that can be different for each realization, based on four stochastic parameters: the gully slope exponent,  $b$ , angles of repose of the gully and fan, and the distance from the ridge of the cap to the initial point of the gully. The model checks to see if the gully is deep enough to get into the waste. If it is, then waste material is assumed to cover the surface area of the fan, and the surface area of the exposed waste is calculated. To simplify the calculation, waste concentrations are averaged over the waste layers exposed and then assigned to an exposure area that corresponds to the surface area of the fan plus the area of the waste exposed within the gully.

A random number of gullies sampled from a discrete distribution (min = 1, max = 20) are chosen to occur, to illustrate the effect multiple gullies would have on dose and to evaluate the effects of gullies over a range of more than one gully. Each of these gullies is identical for a given realization, to keep the gully model simple. The fraction of the cap's surface area that is consumed by gullies is calculated as a reality check to determine if the quantity of erosion is physically reasonable for an intact embankment.

## 6.1 Numerical Solution in GoldSim

GoldSim allows the user to iteratively solve a system of equations, such as what is given above, using Newton's method. This numerical solution is implemented in GoldSim using a previous value element and a looping container for which the user specifies a maximum number of loop counts and/or a convergence criterion.

Newton's method is a successive approximation method that can be used on differentiable functions. In this model, it is the height of the gully mouth that is the function of interest. The formula for Newton's method in terms of gully height is:

$$h_{n+1} = h_n - \frac{f(h_n)}{f'(h_n)} \quad (38)$$

where

- $f(h)$  is the difference between the volume of the gully and the volume of the fan, and
- $f'(h)$  is the derivative of the function  $f(h)$ .

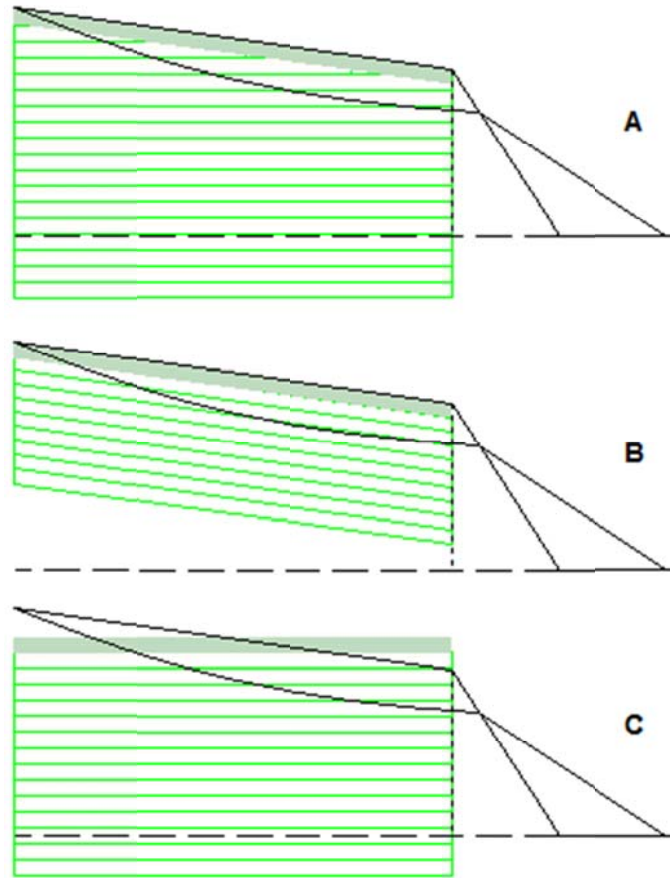
New values are calculated for  $h$  until the difference between  $h_{n+1}$  and  $h_n$  is sufficiently small. In the Clive PA model, a convergence criterion of  $0.01 \text{ m}^3$  is used, such that the difference between the volume of the gully and the volume of the fan is less than  $0.01 \text{ m}^3$ .

## 6.2 Representation of Gully and Waste

The biggest concern about gullies is whether or not a gully gets deep enough to expose and remove waste and how much waste is exposed and removed. In the current Clive PA model, waste is buried only under the top slope, so the quantity of concern is the distance from the ridge that the gully gets into the waste. In similar terminology to that used above, this variable can be called  $L_{gully}$ , where  $L_{gully}$  is a vector of distances from the ridge of the cap to where the gully enters the waste layer.

Some assumptions need to be made to allow for a simple calculation of  $L_{gully}$ . The column of waste and cap, as modeled in the Clive PA model, is a 1-dimensional representation of the cap; however, the gully model is a 2-dimensional representation, to include the slope of the cap in the gully calculations. To calculate  $L_{gully}$  and the gully outwash of each waste layer, the 2-D representation must be merged with the 1-D representation. Figure 6 illustrates the potential configurations that were considered in the calculation of gully outwash volume and the calculate

of where the gully first gets into the waste. The top slope has waste layers (outlined in green) with a cap covering the waste layers. The side slope comes off the top slope, with the break in slope being the vertical dotted black line. The fan leans against the side slope on the right side of the illustrations in Figure 6. The gully (drawn in black) intersects the cap and waste layers.



**Figure 6. Gully and waste configurations for the gully outwash volume calculation, with the cover represented by the solid green layer and the waste layers outlined in green. A. the embankment as constructed; B. representation of the 1-D column, preserving the slope of the cap throughout the column; C. representation of the 1-D column, preserving the horizontal layering of the waste.**

Three different representations were considered to model the intersection of waste with the gully and the gully outwash volume of waste. Figure 6A roughly depicts the embankment as it is intended to be constructed, with the cover (solid green) over horizontal waste layers and a sloping top cap. This representation is difficult to model in GoldSim because the waste layers do not continue across the entire top slope of the cap. Figure 6B represents the waste layers continuing for the length of the cap, at the same slope of the top slope of the cap. The problem with this approach is that the gully can dip into and out of a waste layer, meaning that there are two points in each waste layer that the gully could potentially intersect, rather than one intersection point. With this arrangement, there is considerable computational effort required to ensure that the numerical solution for the volume from each layer converges. Figure 6C shows how the implementation of the gully was chosen for the Clive PA model. The top of the cap and top of

each waste layer is set as the midpoint of each layer in the top slope of the cap. With the horizontal waste layers, the gully intersects each layer only once.

A problem with implementation of the arrangement in Figure 6C is that the top waste layers at the break in slope in this 1-D representation are higher than the actual cap height at the break in the 2-D representation. So the calculation is an approximation of how many waste layers are cut into by the gully and how much waste is washed out by the gully from each waste layer. This approximation is considered tolerable since the overall gully model is a simplification. Furthermore, if there is sufficient fill material between the top of the DU waste and the bottom of the cap, then the effect is negligible. Some caution should be exercised when interpreting output from the gully model if DU waste is disposed within a few meters of the bottom of the cap.

### 6.3 Calculation of $L_{gully}$ .

To calculate the volume of each waste layer removed by the gully and the surface area of the waste layers exposed by the gully, the distance from the ridge of the top slope to where the gully first intersects each waste layer,  $L_{gully}$ , must be calculated. These values are calculated by finding the intersection of the gully with the horizontal lines at the heights above ground surface for each waste layer. In other words, solve Eq. (9) for  $L$  such that  $z_{gully}$  equals the height of each waste layer,  $z_{waste}$ :

$$L_{gully} = \left( \frac{b+1}{a} (z_{waste} + B_1 a - B_0) \right)^{\frac{1}{b+1}} \quad (39)$$

where

$z_{waste}$  is a vector of the waste layer heights above ground surface at the midpoint of the top slope of the cap

Note that this calculation in GoldSim requires that  $B_1 a - B_0$  is a vector expression of length equal to the number of waste layers.

### 6.4 Calculation of Surface Area

The surface area of the fan and the surface area of the waste exposed by the gully will be summed and included in exposure area calculations in the dose assessment.

#### 6.4.1 Surface Area of Fan

To approximate the surface area of the fan, a simplifying assumption is used – that the surface area of the fan is the shadow the fan creates on the horizontal plane. This assumption is reasonable since the fan is at such a low angle (5 – 10 degrees, see Section 4.1). Figure 5 shows the shape of this shadow.

The area of a circular sector,  $Area_{sector}$ , can be found by



$$Area_{sector} = \frac{1}{2} R^2 \theta \quad (40)$$

where

- $R$  is the radius of the circle (m), and
- $\theta$  is the angle cut by the circular segment.

The value of  $R$  is the same as in Eq. (31) above. The value of  $\theta$  is given by

$$\theta = 2 \cos^{-1} \left( \frac{r}{R} \right) = 2 \cos^{-1} \left( \frac{\frac{h}{\tan \alpha_{SS}}}{\frac{h}{\tan \alpha_{fan}}} \right) = 2 \cos^{-1} \left( \frac{\tan \alpha_{fan}}{\tan \alpha_{SS}} \right) \quad (41)$$

where

- $r$  is the horizontal distance from the gully mouth to the outer edge of the side slope, as given in Eq. (32) above.

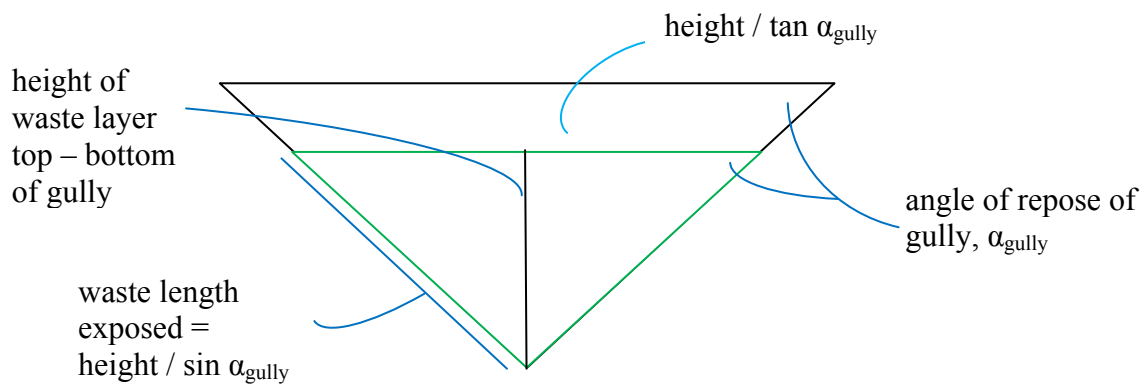
Thus, the surface area of the fan,  $SA_{fan}$ , can be expressed as

$$SA_{fan} = \frac{h^2}{\tan^2 \alpha_{fan}} \cos^{-1} \left( \frac{\tan \alpha_{fan}}{\tan \alpha_{SS}} \right) \quad (42)$$

For more information on understanding this area calculation, see Weisstein, 2011b, for example.

#### 6.4.2 Surface Area of Waste Exposed by Gully

To calculate the surface area of each waste layer exposed, the cross sectional distance of waste exposed by the gully is integrated over the length of the gully in the top slope. See Figure 7.



**Figure 7. Gully cross section for waste exposure calculations.**

The surface area exposed by the gully for each waste layer can be calculated by first calculating the surface area exposed by the gully from the top of each waste layer to the bottom of the gully and then subtracting that calculation from each waste layer. In other words,

$$SA_{WasteLayer1} = SA_{WasteLayer1toGullyBottom} - SA_{WasteLayer2toGullyBottom} \quad (43)$$

So the surface area exposed from the top of each waste layer to the gully bottom is:

$$SA = \frac{2}{\sin \alpha_{gully}} \int_{L_{waste}}^{L_{TS}} (z_{waste} - z_{gully}) dL \quad (44)$$

where the 2 comes from having two sides of the gully exposed.

Substituting in for  $z_{gully}$

$$SA = \frac{2}{\sin \alpha_{gully}} \int_{L_{waste}}^{L_{TS}} (z_{waste} - (\frac{a}{b+1} L^{b+1} - B_1 a + B_0)) dL \quad (45)$$

Simplifying yields

$$SA = \frac{2}{\sin \alpha_{gully}} \left[ (z_{waste} + B_1 a - B_0)(L_{TS} - L_{waste}) - \frac{a}{(b+1)(b+2)} (L_{TS}^{b+2} - L_{waste}^{b+2}) \right] \quad (46)$$

This value of surface area is then evaluated for each waste layer and used as in Eq. (43) to calculate the surface area exposed for each waste layer. The bottom waste layer surface area is simply the value given in Eq. (46).

## 6.5 Calculation of Volume of Waste Layers Removed

In a similar fashion to the calculation for the surface area of waste exposed by the gully, the volume of each waste layer removed by the gully is calculated by first calculating the volume of waste removed from the top of each waste layer to the bottom of the gully. Then that volume calculation is subtracted from the layer below, similar to Eq. (43):

$$Vol_{WasteLayer1} = Vol_{WasteLayer1toGullyBottom} - Vol_{WasteLayer2toGullyBottom} \quad (47)$$

The cross-sectional area of the waste exposed by the gully, similar to Eq. (14), is integrated over the length of the gully that incises the waste.

$$Vol = \frac{1}{\tan \alpha_{gully}} \int_{L_{waste}}^{L_{TS}} (z_{waste} - z_{gully})^2 dL \quad (48)$$

Simplifying,

$$Vol = \frac{1}{\tan \alpha_{gully}} \int_{L_{waste}}^{L_{TS}} (z_{waste}^2 - 2z_{waste} z_{gully} + z_{gully}^2) dL \quad (49)$$

$$Vol = \frac{1}{\tan \alpha_{gully}} \int_{L_{waste}}^{L_{TS}} (z_{waste}^2 - 2z_{waste} \left( \frac{a}{b+1} L^{b+1} - B_1 a + B_0 \right) - \frac{a^2}{(b+1)^2} L^{2b+2} + \frac{2aL^{b+1}}{b+1} (B_0 - B_1 a) + (B_0 - B_1 a)^2) dL \quad (50)$$

$$Vol = \frac{1}{\tan \alpha_{gully}} \int_{L_{waste}}^{L_{TS}} (z_{waste}^2 + 2z_{waste} B_1 a - 2z_{waste} B_0 + (B_0 - B_1 a)^2) + \left( -\frac{2z_{waste} a}{b+1} L^{b+1} + \frac{2a(B_0 - B_1 a)}{b+1} L^{b+1} + \frac{a^2}{(b+1)^2} L^{2b+2} \right) dL \quad (51)$$

$$Vol = \frac{1}{\tan \alpha_{gully}} \left[ (z_{waste}^2 + 2z_{waste} B_1 a - 2z_{waste} B_0 + (B_0 - B_1 a)^2) (L_{TS} - L_{waste}) + \left( \frac{2z_{waste} a}{(b+1)(b+2)} + \frac{2a(B_0 - B_1 a)}{(b+1)(b+2)} \right) (L_{TS}^{b+2} - L_{waste}^{b+2}) + \frac{a^2}{(b+1)^2 (2b+3)} (L_{TS}^{2b+3} - L_{waste}^{2b+3}) \right] \quad (52)$$

This calculation of volume is then evaluated for each waste layer and used as in Eq. (47) to calculate the volume of waste removed by the gully for each waste layer. The bottom waste layer volume is simply that value given in Eq. (52) evaluated for the last waste layer.

## 6.6 Concentration of Waste Removed by Gully

The concentration of waste removed by the gully is averaged and is assumed to be spread out uniformly over the surface area of the fan. This same averaged concentration of waste is assumed to be present in the surface area exposed by the gully.

To obtain the average waste concentration, the concentration of each radionuclide species is computed as a mass-weighted average. The volume of each layer of waste removed by the gully is multiplied by the bulk density of that waste layer to get the mass of waste removed in each layer. Then the mass in each layer is divided by the total mass of waste removed. The mass of each radionuclide in each waste cell is converted to a mass concentration and then multiplied by the mass fraction of each layer removed by the gully. The concentration of waste removed by the gully is then the sum of each radionuclide over every waste layer. It is this total concentration that is used in the dose calculations.

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